Quantum damping of position due to energy measurements

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(February 1, 2008)

Abstract

Quantum theory for measurements of energy is introduced and its consequences for the average position of monitored dynamical systems are analyzed. It turns out that energy measurements lead to a localization of the expectation values of other observables. This is manifested, in the case of position, as a damping of the motion without classical analogue. Quantum damping of position for an atom bouncing on a reflecting surface in presence of a homogeneous gravitational field is dealt in detail and the connection with an experiment already performed in the classical regime is studied. We show that quantum damping is testable provided that the same measurement strength obtained in the experimental verification of the quantum Zeno effect in atomic spectroscopy [W. M. Itano et al., Phys. Rev. A **41**, 2295 (1990)] is made available.

03.65.Bz, 42.50.Wm

I. INTRODUCTION

A number of new experimental techniques involving devices with noise figures close to the minimum dictated by the uncertainty principle has originated the demand for quantitative predictions of quantum measurement theory [1,2]. One of the most important steps along this path was made by Itano et al. [3]: they experimentally showed that the effect of the measurement on the observed system leads to a freezing of its free dynamical evolution, the so-called quantum Zeno effect [4,5]. In such a case one is looking at the occupancy probability of one level, that happens to depend upon the process of measurement itself. More general quantum measurement effects can be obtained by measuring one observable and by looking at the dynamical evolution of another observable, as it has been already discussed for a mesoscopic structure in [6]. This agrees with the original spirit with which Sudarshan opened the debate on the quantum Zeno effect and according to which measurement of the position of a decaying particle should influence its lifetime, i.e. another observable quantity.

In this paper we discuss a model for a system subjected to quantum measurements of energy including their influence on other observables, specifically position measurements. The act of measurement affects the average position of the system giving rise to an average localization of the motion, a *quantum damping* without classical counterpart. The paper is organized as follows. In Section II we introduce the general formalism for measurements of energy distinguishing between nonselective and selective cases and we evaluate the average position of the measured system in both schemes. The example of the harmonic oscillator, for which analytical evaluations are possible, is also dealt. In Section III a particle in a homogeneous gravitational field bouncing on a perfectly reflecting surface is analyzed in detail. In particular, the dependence of the quantum localization effect upon the relevant parameters is studied both by means of numerical tools as well as in the semiclassical limit, by exploiting the WKB approximation. In Section IV possible scenarios to look for the predicted quantum damping are discussed with particular emphasis on the previously considered particle bouncing on a reflecting surface. We focus on a case very close to the

experimental situation already reported in [7], giving a discussion of the parameters that have to be reached to look for the predicted effect.

II. QUANTUM MEASUREMENTS OF ENERGY: GENERAL FORMALISM

Since the original proposal of Von Neumann [8], important progresses have been made in understanding quantum measurement theory. In particular, it has been recognized that the dynamics of the system in presence of the measurement undergoes modifications, with respect to the closed system dynamics, that can be taken into account by means of an effective master equation or a Schrödinger equation for mixed and pure states respectively. Thus the original doubling of dynamics, a free Schrödinger evolution during the non measurement periods and an abrupt state collapse during an ideal, instantaneous measurement, has been replaced by a unique dynamical approach. In this context, some models represent the meter as a particular environment that interacts with the observed system and extracts information from it. The dynamics of a system interacting with an environment is most conveniently described in terms of a reduced density matrix operator $\hat{\rho}(t)$, obtained by tracing out the degrees of freedom of the environment from the density matrix operator of the entire system (marginalization procedure). The unitary evolution of $\hat{\rho}(t)$ for the isolated system is modified to an irreversible one due to the interaction with the environment. In the limit of a Markovian environment, a dynamical law described by a completely positive semigroup can be postulated for the evolution of the open system [9,10], resulting in the following master equation for the reduced density matrix operator:

$$\frac{d}{dt}\,\hat{\rho}(t) = -\frac{i}{\hbar}\left[\hat{H}(t), \hat{\rho}(t)\right] + \frac{1}{2}\sum_{\nu=1}^{n}\left(\left[\hat{L}_\nu(t)\hat{\rho}(t), \hat{L}_\nu(t)^\dagger\right] + \left[\hat{L}_\nu(t), \hat{\rho}(t)\hat{L}_\nu(t)^\dagger\right]\right) \qquad (1)$$

where $\hat{H}(t) = \hat{H}(\hat{p}, \hat{q}, t)$ is the Hamilton operator for a general nonautonomous system and $\hat{L}_\nu(t)$, $\nu = 1, \ldots, n$, are the so-called *Lindblad operators*, that are supposed to model the effects of the environment on the system. The above equation describes the general case of a quantum open system. The evolution of a quantum system subjected to a measurement process corresponds to the particular case where the environment is the measurement apparatus

and the Lindblad operators are proportional to the measured quantities. If the measurement of a single observable represented by the operator $\hat{A}(t) = \hat{A}(\hat{p}, \hat{q}, t)$ is considered, the corresponding Hermitian Lindblad operator can be chosen to be $\hat{L}(t) \equiv \hat{L}(t)^\dagger = \sqrt{\kappa(t)}\hat{A}(t)$. The function $\kappa(t)$ has dimensions $[\kappa] = [t^{-1}A^{-2}]$ and represents the coupling, in general time-dependent, of the monitored system to the measurement apparatus. The result of the measurement is

$$\overline{a(t)} = \mathrm{Tr}\left(\hat{A}(t)\hat{\rho}(t)\right) \ , \tag{2}$$

where overlining denotes a statistical average over individual results associated to pure states that form the incoherent mixture described by the reduced density matrix operator $\hat{\rho}(t)$.

By moving to the coordinate representation of the reduced density matrix operator,

$$\rho(q_1, q_2, t) = \langle q_1 |\ \hat{\rho}(t)\ | q_2 \rangle \ , \tag{3}$$

the evolution equation obtained from (1) is

$$\begin{aligned}\frac{\partial}{\partial t}\,\rho(q_1, q_2, t) = \Bigg\{ &-\frac{i}{\hbar}H\left(-i\hbar\frac{\partial}{\partial q_1}, q_1, t\right) + \frac{i}{\hbar}H\left(-i\hbar\frac{\partial}{\partial q_2}, q_2, t\right) \\ &-\frac{1}{2}\kappa(t)\left[A\left(-i\hbar\frac{\partial}{\partial q_1}, q_1, t\right) - A\left(-i\hbar\frac{\partial}{\partial q_2}, q_2, t\right)\right]^2\Bigg\}\rho(q_1, q_2, t) \ . \end{aligned} \tag{4}$$

Equation (4) gives the general description of a system in which the observable $\hat{A}$ is continuously monitored and the result of the measurement is not known in advance, the so-called *nonselective* measurement process.

If continuous measurements of energy characterized by a constant coupling κ_E are considered ($\hat{A} = \hat{H}$), Eq. (4) for the evolution of the density matrix specializes to

$$\dot{\rho}_{nm} = -\frac{i}{\hbar}(E_n - E_m)\rho_{nm} - \frac{\kappa_E}{2}(E_n - E_m)^2\rho_{nm} \ , \tag{5}$$

where

$$\rho_{nm}(t) = \int\int\ dq\ dq'\ \rho(q, q', t)\phi_n^*(q)\phi_m(q') \tag{6}$$

and $\{\phi_n\}$ is the complete set of energy eigenstates, corresponding to the energy eigenvalues E_n. It is easy to show that the solution of the equation (5) can be written as

$$\rho_{nm}(t) = \exp\left\{-\frac{i}{\hbar}(E_n - E_m)t - \frac{\kappa_E}{2}(E_n - E_m)^2 t\right\} \rho_{nm}(0) \ , \tag{7}$$

and consequently

$$\rho(q, q', t) = \sum_{nm} \rho_{nm}(t) \phi_m^*(q') \phi_n(q) \ . \tag{8}$$

Notice that the effect of the measurement in (7) is to diagonalize the density matrix after enough time, the so-called *decoherence* induced by the measurement [11].

Without loss of generality, we can restrict our analysis to the position operator $\hat{Q}$. The average position at time t is then evaluated as

$$\langle Q(t) \rangle = \int dq \ dq' \ \rho(q, q', t) \ \langle q' | \hat{Q} | q \rangle = \int dq \ \rho(q, q, t) \ q \tag{9}$$

and therefore from (8)

$$\langle Q(t) \rangle = \sum_{nm} \rho_{nm}(t) \langle n | \hat{Q} | m \rangle \ . \tag{10}$$

By inserting (7), the average position in the nonselective case we have just considered is finally written as

$$\langle Q(t) \rangle = \sum_{nm} \exp\left\{-\frac{i}{\hbar}(E_n - E_m)t - \frac{\kappa_E}{2}(E_n - E_m)^2 t\right\} \rho_{nm}(0) \langle n | \hat{Q} | m \rangle \ , \tag{11}$$

where the effect of the measurement leads to exponential decaying behaviour with state-dependent time constants

$$\tau_{nm} = \frac{2}{\kappa_E (E_n - E_m)^2}. \tag{12}$$

It is worth to observe that the off-diagonal terms of the sum contributing to the average position vanish in the asymptotic limit $t \to \infty$ when $\kappa_E > 0$. As a consequence, the average position tends to be localized around the stationary value due to the diagonal contributions. From the conceptual viewpoint, this effect is nothing but the manifestation, in the configuration space, of the previously mentioned decoherentization process.

Another interesting situation arises when the energy measurement is already performed and the result is known. The theory should be able, in this case, to complete the knowledge of

the system by evaluating the corresponding wavefunction conditioned to the known result of the energy measurement, the so-called *a posteriori selective* measurement. In [12] it is shown that, starting from (4), an effective Schrödinger equation for an *a posteriori* measurement is obtained for the restricted wavefunction $\psi_{[a]}(q,t)$ as:

$$i\hbar\,\frac{\partial\psi_{[a]}(q,t)}{\partial t} = \left[H(-i\hbar\frac{\partial}{\partial q},q,t) - i\hbar\,\kappa_{[a]}(t)\left[A(-i\hbar\frac{\partial}{\partial q},q,t) - a(t)\right]^2\right]\psi_{[a]}(q,t)\;, \qquad (13)$$

where we have denoted by a the result of the measurement and by $[a]$ the associated functional dependence. Equation (13) formally holds also when the result $a(t)$ is unknown and has to be predicted with probability distribution $\|\psi_{[a]}(t)\|^2$ (*a priori selective measurement*); a different interpretation of (13) is however required, for which we refer to [12]. Coming back to continuous measurements of energy with a constant result $E(t) = E$, Eq. (13) has the solution [6,13]

$$\psi_{[E]}(q,t) = \sum_n c_n(0)\exp\left\{-iE_nt/\hbar - \kappa_E(E_n-E)^2t\right\}\,\phi_n(q)\;. \qquad (14)$$

Here, as before, $\phi_n(q)$ are energy eigenfunctions, $c_n(0)$ the corresponding initial projection coefficients, and normalization to unity is lost due to the non Hermitian character of Eq. (13), that expresses in more physical terms the branching of the wavefunction among all the possible alternatives. By exploiting (14), the average position in this selective case, hereafter denoted by $\langle Q(t)\rangle_{[E]}$, is therefore written as

$$\langle\, Q(t)\rangle_{[E]} = \langle\psi_{[E]}|\hat{Q}|\psi_{[E]}\rangle/\langle\psi_{[E]}|\psi_{[E]}\rangle = \left[\sum_k |c_k(0)|^2\exp\{-2\kappa_E(E_k-E)^2\,t\}\right]^{-1}\cdot$$
$$\cdot\;\sum_{nm} c_n(0)c_m^*(0)\exp\left\{-i(E_n-E_m)t/\hbar - \kappa_E[(E_n-E)^2+(E_m-E)^2]\,t\right\}\,\langle m|\hat{Q}|n\rangle\;. \qquad (15)$$

The link between the average position in the nonselective (11) and selective (15) cases is established, in the usual way [12], by summing over all the possible selective measurement processes, i.e.

$$\rho(q,q',t) = \int\; d[E]\;\psi^*_{[E]}(q,t)\;\psi_{[E]}(q',t)\;, \qquad (16)$$

that also shows how the nonselective measurement, described in terms of the density matrix, can be seen as a functional integration over all the possible selective measurements,

represented by the restricted wavefunction. Note also that the decay time constants in (15) explicitly depend upon the value E registered during the measurement.

The theoretical scheme we have presented can be easily implemented for a simple system as the harmonic oscillator. Let us denote by m and ω the mass and the angular frequency respectively. The effect of the measurement on the average position in the nonselective case can be guessed by simple inspection of Eq. (11). Indeed, due to the equal spacing between adiacent levels, one immediately realizes that the measurement damping effect factors out and acts on the remaining unmeasured evolution as a purely exponential damping with an unique time constant $\tau = 2/\kappa_E \hbar^2 \omega^2$, that is:

$$\langle Q(t) \rangle_{\kappa_E} = \exp\left\{ -\frac{\kappa_E (\hbar\omega)^2 t}{2} \right\} \, \langle Q(t) \rangle_{\kappa_E = 0} \, . \tag{17}$$

In the selective case instead the corresponding expression (14) can be explicitly computed if, as usual, creation and annihilation operators $\hat{a}, \hat{a}^\dagger$ are introduced so that

$$\hat{Q} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger) \, . \tag{18}$$

Then, owing to the well known selection rule for the matrix elements of the position operator between energy eigenstates, namely

$$\langle n|\hat{Q}|m\rangle = \sqrt{\frac{\hbar}{2m\omega}} \, \langle n|(\hat{a} + \hat{a}^\dagger)|m\rangle = \sqrt{\frac{\hbar}{2m\omega}} \, (\sqrt{m} \, \delta_{n,m-1} + \sqrt{m+1} \, \delta_{n,m+1}) \, , \tag{19}$$

the following expression for the average position is found:

$$\langle Q(t) \rangle_{[E]} = 2 \left(\frac{\hbar}{2m\omega} \right)^{1/2} \left\{ \sum_k |c_k(0)|^2 \exp[-2\kappa_E (E_k - E)^2 t] \right\}^{-1} \cdot \sum_{n=0}^{\infty} \sqrt{n+1} |c_n| |c_{n+1}| \cdot \tag{20}$$

$$\cdot \exp\left\{ -2\kappa_E \left[\left[\left(n + \frac{1}{2} \right) \hbar\omega - E \right]^2 + \hbar\omega \left[\left(n + \frac{1}{2} \right) \hbar\omega - E \right] + \frac{\hbar^2 \omega^2}{2} \right] t \right\} \cos(\omega t + \theta) \, ,$$

where θ is the relative phase between c_n and c_{n+1}. Unlike the nonselective case, no factorization of the damping factor is allowed here. By referring to the decay constant for the nonselective measurement τ, we get

$$\tau_{[E]}(n) = \frac{\tau}{4} \left[\left(n + \frac{1}{2} - \frac{E}{\hbar\omega} \right)^2 + n + 1 - \frac{E}{\hbar\omega} \right]^{-1} \, . \tag{21}$$

The time constants will now depend upon the registered energy E, decreasing with the difference between the ratio $E/\hbar\omega$ and the average number of quanta in the state.

III. FREE FALL OF A PARTICLE AND BOUNCING ON A REFLECTING SURFACE

Another stimulating example is the monitoring of the energy of a particle falling in an homogeneous gravitational field and bouncing on an elastically reflecting surface. Experiments already performed have shown that multiple bouncing of atoms on surfaces is possible [7] and the classical sources of damping have been understood [14]. The experiment has been performed by measuring the average of an ensemble of independent atoms and therefore the nonselective approach described in Section II is more adequate.

Let us consider the height z over the surface and the potential energy

$$V(z) = \begin{cases} m\,g\,z & z > 0\ , \\ +\infty & z \leq 0\ . \end{cases} \tag{22}$$

The Schrödinger equation for positive z and energy eigenvalue E is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2} + (mgz - E)\psi = 0 \tag{23}$$

and we assume, consistently with the boundary conditions of the problem, $\psi(z) = 0$ for $z \leq 0$. Equation (23) is exactly solvable and the solutions are expressed in terms of the Airy functions [15]

$$\phi_n(z) = C_n\ Ai\,(z/z_0 - \lambda_n)\ , \tag{24}$$

where we have introduced the characteristic length of the system

$$z_0 = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3}\ . \tag{25}$$

The C_n's are normalization constants and $-\lambda_n$ is the n-th zero of the Airy function, related to the corresponding energy eigenvalue through the following relation:

$$E_n = \frac{\hbar^2}{2mz_0^2}\,\lambda_n\ . \tag{26}$$

According to these conventions, $z_0\lambda_n$ represents the n-th classical turning point, also denoted by z_n.

The distance between consecutive zeroes of the Airy functions $|\lambda_n - \lambda_{n+1}| \to 0$ for $n \to \infty$. It is therefore recognizable that, for this system, unlike the harmonic oscillator, the quantum damping is large for states formed by low-energy pairs of consecutive eigenstates and vanishes in the (classical) limit of high energy. To understand the mechanism of damping we have numerically studied various cases corresponding to different initial preparations of the system. The numerical accuracy of the program, tested with the already analytically solved harmonic oscillator, is of the order of 0.1% for reasonable values of the space-time lattice. If energy eigenstates are considered, the average position is constant in time but, unlike the case of the harmonic oscillator, it is different from zero due to the nonvanishing diagonal matrix elements of the position operator. However, these stationary states are not affected by the presence of a measurement coupling. On the other hand, when a superposition of two energy eigenstates is assigned at zero time, the average position will harmonically oscillates between a minimum and a maximum value with angular frequency $\omega = (E_n - E_m)/\hbar$ ($n = 2, m = 1$ in the example of Fig. 1a). By comparison, the constant values of the average position in the two energy eigenstates are also shown, the quantities

$$Q_{11} = \int dz\, \phi_1(z)^*\, z\, \phi_1(z) \quad ; \quad Q_{22} = \int dz\, \phi_2(z)^*\, z\, \phi_2(z)\ , \tag{27}$$

which allows one to write the average position of their superposition $\psi = c_1\phi_1 + c_2\phi_2$, as

$$\langle Q(t)\rangle = |c_1|^2\ Q_{11} + |c_2|^2\ Q_{22} + 2|c_1||c_2|Q_{12}\cos\left[\frac{(E_1 - E_2)t}{\hbar} + \theta\right]\ , \tag{28}$$

with

$$Q_{12} = \int dz\, \phi_2(z)^*\ z\ \phi_1(z) = Q_{21}^*\ . \tag{29}$$

In presence of a measurement a pure exponential damping arises that still reminds the oscillatory behaviour (as shown in Fig. 1b) or that is overdamped (as in Fig. 1c), depending upon the measurement coupling through the time constant $\tau_{12} = 2\,[\kappa_E(E_2 - E_1)^2]^{-1}$. The time development is centered around the mean value of the unmeasured evolution (28). The effect of the energy measurement vanishes in the limit of large quantum numbers, namely

the classical limit. This can be easily shown by exploiting the WKB approximation, that gives the spacing between two consecutive levels as reported in [16]

$$E_n - E_{n-1} = \pi\hbar \left(\frac{g}{2z_n}\right)^{1/2} . \tag{30}$$

Since it is difficult to obtain superpositions of only two energy eigenstates, the attention must be focused on states that are more likely to be produced when an atomic cloud is prepared. This is the case of the Gaussian states, already extensively studied for being the quantum states closest to classicality, although still far from representing an atomic cloud. As discussed in [17], Cesium atoms in magnetooptical traps lead at best to an initial radius of the cloud equal to $z_i \simeq 50\ \mu$m with an rms momentum corresponding to a velocity spread of $\simeq 2\,$cm/s, leading to an uncertainty product $z_i p_{z_i} \simeq 2000\,\hbar$. Nevertheless, Gaussian states constitute entities simple enough to be analyzed, allowing a physical understanding of their behaviour, and at the same time complex enough to maintain the typical features of the more realistic situations. Indeed, the striking difference from the already analyzed case is that now the energy eigenstates expansion cointains different eigenstates and a more complex dynamics is obtained also in the unmeasured case (Fig. 2a). When the effect of the measurement is taken into account, a damping of the motion is obtained in all the explored cases, although it is not a pure exponential damping, as shown from the weak reformation of the damped oscillations in Fig. 2b. By varying the height of the atomic center of mass for a Gaussian state the relevant contributing eigenstates also change. In general, as also intuitively understandable, for a height z the greatest contributions will stem from the eigenstates having the classical turning point $z_0\lambda_n$ closest to z. Moreover, the number of contributing eigenstates will depend upon the width of the initial Gaussian state. In Fig. 3 a configuration similar to Fig. 2 but with a larger width of the Gaussian state is depicted. In this case, more energy eigenstates significantly contribute to the expansion and in particular the lower ones determine, due to their larger energy difference, a faster dynamics for the average position. As a general feature, among the various time constants contributing to the damping, the more relevant ones correspond to the eigenstates that at

the same time possess larger energy separation and appreciable contribution to the state itself.

Also here the WKB approximation allows one to estimate the damping time constants. By considering a rough picture in which a state is only made of eigenstates centered around $\bar{n}$ with width $2\,\Delta\bar{n}$, all contributing with the same weight, the dynamics is ruled by the farest energy eigenvalues:

$$\begin{aligned}\tau_{min} &= \frac{2}{\kappa_E |E_{\bar{n}+\Delta\bar{n}} - E_{\bar{n}-\Delta\bar{n}}|^2} \\ &= \left(\frac{2}{3\pi}\right)^{4/3} \frac{8m^2 {z_0}^4}{\kappa_E \hbar^4} \left[\left(\bar{n} + \Delta\bar{n} - \frac{1}{4}\right)^{2/3} - \left(\bar{n} - \Delta\bar{n} - \frac{1}{4}\right)^{2/3}\right]^{-2} . \end{aligned} \tag{31}$$

IV. PHENOMENOLOGICAL CONSIDERATIONS

In this section we analyze observable consequence of the measurement of energy in the two cases of harmonic oscillators and bouncing particles in the presence of gravitational fields. Before doing this, some preliminar considerations are needed about the relevant coupling constant of the model, κ_E. Bearing in mind that this parameter, describing the effective coupling of the apparatus to the observed system, intrinsically depends upon the particular experimental setup, one can only assess for it a reasonable value. If the analysis for a continuous nonselective measurement of energy is applied to the results of Itano et al. as done in [12], a lower bound for the coupling parameter κ_E is found. In [12] the coupling parameter is expressed in terms of a critical value $\kappa_{crit} = 4\omega_R$, where $\omega_R = 12.272 s^{-1}$ is the Rabi angular frequency (corresponding to a period of the radiofrequency of $256\,ms$) and the energy difference between the two levels $E_2 - E_1 = 2.125 \cdot 10^{-25} J$. The data of the experiment are fitted for $\kappa_E^{exp} \approx 10^2 \kappa_{crit}/(E_2 - E_1)^2 = 10^{53}\, J^{-2} s^{-1}$. This value will be assumed, in some of the following examples, as an indicative one.

It is worth to note that, due to the generality of the model, observable effects are in principle expected in all physical situations where quantum measurements are involved.

Among all the possible systems in which the effect can be made observable, we have chosen to discuss atomic or molecular systems and single degrees of freedom of macroscopic bodies.

Let us first reconsider the case of systems modelizable as harmonic oscillators. If, for example, the vibrational energy levels of a biatomic molecule are monitored and the position operator $\hat{Q}(t)$ is interpreted as describing the istantaneous electric dipole moment of the molecule along the internuclear axis, then, according to (17), one can roughly expects a modification of the law according to which emission and absorption of electromagnetic radiation occurs. For instance, the intensity $I(t)$ will not decay with an exponential law ruled by a lifetime τ, but it will manifest a more complex behaviour:

$$I(t) = I_0 \exp\Big\{ - \exp\left[\kappa_E (\hbar\omega)^2 t\right] \frac{t}{\tau} \Big\} \tag{32}$$

corresponding to an inhibition of the decay. Unfortunately, the time scale resulting when the κ_E^{exp} quoted before is of the order of $10^8 s$ for reasonable vibrational frequencies, prohibitively long compared to the typical lifetimes of the vibrational transitions. Other perspectives can be opened by considering the time and frequency resolved spectra of spontaneous emission recently demonstrated [18].

Alternatively one could measure the energy in mechanical harmonic oscillators such as the resonators used as gravitational wave antennas, provided that the quantum limit is achieved in such a class of detectors [19,20]. Due to the presence of a single system the analysis should be carried out by using the selective measurements approach. In this case, however, one big problem is the difficulty to obtain electromechanical transducers that measure the energy of the oscillator with enough sensitivity [21].

Other possibilities are also open by exploiting the quantum measurement model applied to the bouncing particle of Section III. An experiment aimed at testing the quantum damping can be designed on the basis of already performed experiments as the ones described in [3] and in [7]. A cloud of atoms is trapped and cooled at a given height over a dielectric surface as in [7]. Along the vertical path we put both a inhomogeneous magnetic field with constant gradient and a radiofrequency. The magnetic field is such that at the initial height the

radiofrequency gives rise to resonant Rabi oscillations between two levels of the atoms. The hyperfine splitting changes with the magnetic field and for each height less than the initial one the resonant condition is not fulfilled. A continuously operating laser acting along the vertical direction is tuned to the optical transition between level 1 and a third level which has forbidden transitions with level 2. A set of optical detectors allows one to observe the fluorescence light proportional to the occupancy of level 1. On the other hand, as shown in [12], the occupancy of a level can be also thought as a measurement of energy since the state projectors of the occupancy and the energy operators coincide apart from a dimensional constant. Due to the spatially variable magnetic field, the continuous measurement of the occupancy will push the state of the atoms toward gravitational energy eigenstates and this will affect their average position. The damping of the average position of the atomic cloud could be measured with a destructive probe photon beam by repeating the measurements many times, as in [7], or by exploiting non-destructive measurement schemes, such as the quantum nondemolition dispersive atomic probe one [22], allowing to repeatedly monitor the same atomic cloud (for more recent proposal see also [23,24]). An estimate of the quantum damping time can be given by using Eq. (31) once expressed in terms of the more accessible position variance of the atomic cloud. The energy spread due to the z-motion of the atomic cloud can be written as

$$\Delta E^2 = \langle H^2 \rangle - \langle H \rangle^2 \,, \tag{33}$$

where $H = p_z^2/2m + V(z)$. By supposing a Gaussian initial atomic phase space distribution both in coordinate and momentum, centered on the z-axis at an height z_E above the reflecting surface, we have a Wigner function

$$W(z, p_z) = \frac{1}{2\pi z_i p_{z_i}} \exp\left[-(z - z_E)^2/2z_i^2\right] \cdot \exp[-p_z^2/2p_{z_i}^2] \tag{34}$$

and the evaluation of the energy spread gives in this case [16]

$$\Delta E = mgz_0\Big[2N^4\Big(\frac{z_0}{z_i}\Big)^4 + \Big(\frac{z_i}{z_0}\Big)^2\Big]^{1/2} \tag{35}$$

where the uncertainty product of coordinate and momentum in units of $\hbar$ has been introduced $z_i p_{z_i}/\hbar = N$. The energy spread has a value equal to $\Delta E \approx mgz_i$ for large values of z_i/z_0, as one expects from the classical behaviour. In the opposite situation the effect of the Heisenberg principle appears and gives an inverse law dependence of the energy width upon the initial position spreading and a branching of the curves for different values of the uncertainty product N. A minimum value of the energy width is obtained for an intermediate value equal to $z_i = 2^{1/3} N^{2/3} z_0$.

The quantum decay constant is expressed as

$$\tau = \frac{2}{\kappa_E \Delta E^2} = \frac{2}{\kappa_E m^2 g^2 z_0^2} \Big[2N^4 \Big(\frac{z_0}{z_i} \Big)^4 + \Big(\frac{z_i}{z_0} \Big)^2 \Big]^{-1} \tag{36}$$

and in correspondence to the minimum of the energy spread gets the maximum value equal to $\tau_{\max}(N) = 2^{-5/3} \kappa_E^{-1} m^{-2} g^{-2} z_0^{-2} N^{-4/3}$.

In Fig. 4 the decay constant versus the ratio z_i/z_0 is shown for various values of the normalized uncertainty product N, including the case of a pure state ($N = 1/2$). Smaller decay times are observed either for large values of the initial position uncertainty, in the right part of all the curves, or in the branching of the curves for the left part, in this last case depending upon the normalized uncertainty product. In the estimate we have assumed a constant width of the atomic cloud, an hypothesis that is not strictly valid due to the spreading following its preparation. To minimize this spreading a very low starting temperature of the atomic cloud is required, for instance ^{85}Rb clouds were shown to double their diameter in 15 ms if cooled at a temperature of about 10μK [25]. Since low temperatures are also associated to small initial diameters of the cloud we expect the approximation of constant diameter to better hold in the left part of the curves drawn in Fig. 4. In these cases values of the quantum damping comparable to the ones of the right part are obtained for widths around ten-one hundred times the fundamental length z_0, i.e. around $2 \div 20\mu$m. For comparison, the point corresponding to the decay observed in [7] is reported. To attribute the damping to the predicted quantum effect the classical sources of damping should be kept as small as possible, due also to the dynamics imposed by the Rabi transition frequency.

Currently achieved values of the escape time of atomic clouds are in the range of 10 seconds [26,27] and if this figure can be maintained together with a measurement coupling of the order of magnitude of the one corresponding to the experiment described in [3] and a normalized uncertainty product $\approx 2 \cdot 10^5$, quantum damping is made observable.

V. CONCLUSIONS

A quantum damping without classical counterpart has been introduced as a consequence of quantum measurements of energy and discussed for two situations. The model of the harmonic oscillator could be implemented in the monitoring of the vibrational motion of biatomic molecules or in quantum limited measurements in macroscopic mechanical resonators. Quantum localization can also be manifested using a cloud of atoms bouncing over a reflecting surface in the presence of a uniform gravitational field. A possible experimental scheme based on this last configuration has been discussed in more detail leading to a proposal that should merge two experiments already separately performed in [3] and [7]. Studies of the quantized structure of particles in gravitational fields and the observation of a damping purely connected to the effect of the measurement as described here give further motivations to improve the cooling capabilities of atomic traps.

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FIGURES

FIG. 1. Average position versus time in the case of a particle bouncing on a reflecting surface for an initial pure state superposition of the first two energy eigenstates with amplitude coefficients $1/2$ and $\sqrt{3}/2$ respectively. The case a) is relative to an unmeasured system ($\kappa_E = 0$) and the dashed curves represent the constant values for the two eigenstates ϕ_1 (below) and ϕ_2 (above). The cases b) and c) are relative to $\kappa_E = 10^{-2}$ and $\kappa_E = 10^{-1}$ representing two examples in the underdamped and overdamped regimes, respectively. Here and in the analogous cases of Figs.2 and 3 we put $\hbar = m = 1$.

FIG. 2. Average position versus time for a particle bouncing on a wall and schematized by a Gaussian state whose center is initially located at $h = 10$ with variance $\sigma = 1$. In the energy eigenstates expansion the main contributions stem from the eigenvalues between the fifth and the nineth. In a) the unmeasured case is depicted, in b) a measured case with $\kappa_E = 10^{-2}$ is shown. Note the persistence of the oscillations which increase after a minimum indicating a not pure exponential damping unlike the one of Fig. 1. The lack of complete periodicity of the unmeasured case here and in the following Fig. 3a is attributable to the presence of many eigenstates contributing to the wavefunction reconstruction on timescales longer than the one depicted.

FIG. 3. The same as in Fig. 2 but for a Gaussian state with variance $\sigma = 3$. The larger spreading corresponds to an increase of the number of eigenstates which significantly contribute to the state, in this case between $n = 3$ and $n = 12$. Case a) is the unmeasured case, case b) is relative to a measurement with $\kappa_E = 10^{-2}$.

FIG. 4. Decay constant for the quantum damping versus the position spreading corresponding to a Gaussian Wigner distribution of Cesium atoms bouncing in a gravitational cavity, normalized to the fundamental gravitational length z_0 (mass $m_{Cs} = 2 \cdot 10^{-25}$Kg, gravitational length $z_0 = 0.23\mu$m). The curves are obtained for different values of the normalized uncertainty product of the atomic cloud, respectively $N = 1/2$ (pure state, a), $N = 20$ (b), $N = 2 \cdot 10^3$ (as experimentally achieved in [17], c), and $N = 2 \cdot 10^5$ (d). For comparison the experimental point explored in [7], corresponding to a decay time of $\simeq 80\,$ms for a value of $z_i/z_0 \approx 10^3$ and explained in terms of classical sources of damping, is shown. It has been assumed a measurement coupling constant $\kappa_E = 10^{53} J^{-2} s^{-1}$.